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FILTRATION IN A POROUS MEDIUM WITH A FLUCTUATING PERMEABILITY

V. V. Dzhabrailov and R. P. Meilanov

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Based on the T-matrix method, filtration in a porous medium with a fluctuating permeability is investigated. A closed system of equations for the T-matrix and the pressure is obtained. A fluctuation correction to the output is calculated. Cases of correlation functions of the first and second kinds are analyzed.

Investigation of the properties of porous media calls for solving several fundamental problems. One of these is associated with the complex spatially inhomogeneous structure, where it is necessary to leave the limits of the usual geometric concepts and invoke notions of fractal structures based on the use of fractional dimensions of space [1-3]. Another problem is associated with the multiphase state of the system and the need to take into account the properties of the phase interface, which is a particular state of the substance; at the present time this problem is being studied extensively and it calls for the development of the statistical physics of strongly nonequilibrium open systems. In porous media the interface portion is such that it contributes to the properties observed [4].

Investigations of the properties of porous media make wide use of methods of development in the quantum statistical physics of disordered media such as the method of percolations the coherent-potential approximation, and various versions of the diagram technique. The use of these methods provides a basis for the stochastic theory of filtration [5]. In the present work we investigate filtration in a porous medium using the T-operator method [6]. On the basis of the equations obtained, we investigate the effect of fluctuations in the permeability on the output. In contrast to the monotonically decreasing correlation functions (of the first kind), used previously we consider correlation functions of another class, namely, nonmonotonous functions (of the second kind).

1. For simplicity of presentation, we consider the case of one-dimensional steady filtration in a stratum with a random permeability [5].The multidimensional case is considered in a similar way. The initial equation has the form

$$\frac{d}{dx}\left[k(x)\frac{d}{dx}P(x)\right]=0\,. \tag{1}$$

At the boundaries of the stratum nonrandom pressures are assigned:

$$P(x=0)=P_1\,,\quad P(x\neq l)=P_2\,, \tag{2}$$

where l is the linear dimension of the stratum. The randomly inhomogeneous permeability $k(x)$ is represented as

$$k(x)=k_0+k_1(x)\,,\quad k_0=\langle k(x)\rangle\,.$$

Here <...> denotes averaging over an ensemble of realizations of the randomly inhomogeneous function $k(x)$; $k_1(x)$ is the fluctuational additive to the permeability. We represent Eq. (1) in the form

$$L_0(x)\,P(x)=-\,L_1(x)\,P(x)\,, \tag{3}$$

where the operators $L_0(x)$ and $L_1(x)$ are defined as follows:

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$$L_0(x) = \frac{d^2}{dx^2}; \quad L_1(x) = \frac{k_1(x)}{k_0} L_0 + \frac{1}{k_0} \frac{dk_1(x)}{dx} \frac{d}{dx}. \tag{4}$$

The solution to Eq. (3) has the form

$$P(x) = P_0(x) + \frac{1}{l} \int_0^l dx_1 G(x, x_1) L_1(x_1) P(x_1), \tag{5}$$

where $P_0(x)$ is the solution to the equation

$$L_0(x) P_0(x) = 0 \tag{6}$$

with boundary conditions (2), and $G(x, x_1)$ is the Green function of the problem (6) and (2):

$$L_0(x) G(x, x_1) = -\delta(x - x_1). \tag{7}$$

We introduce the T-operator according to the relation

$$L_1(x) P(x) = T(x) P_0(x). \tag{8}$$

Substituting Eq. (8) into Eq. (5), we obtain the following expression for the pressure:

$$P(x) = P_0(x) + \frac{1}{l} \int_0^l dx_1 G(x, x_1) T(x_1) P_0(x_1). \tag{9}$$

As follows from Eqs. (8) and (9), the equation of motion for the T-operator can be represented as

$$T(x) P_0(x) = L_1(x) P_0(x) + \frac{1}{l} \int_0^l dx_1 L_1(x) G(x, x_1) T(x_1) P_0(x_1). \tag{10}$$

Using the definitions (4), (7), and (8), it is possible to transform the equation for the T-operator to the form

$$T(x) P_0(x) = \frac{1}{k(x)} \frac{dk_1(x)}{dx} \frac{dP_0(x)}{dx} + \frac{1}{k(x)} \frac{dk_1(x)}{dx} \frac{1}{l} \int_0^l dx_1 G_x(x, x_1) T(x_1) P_0(x_1). \tag{11}$$

Note that the appearance of the multiplier $1/k(x)$ on the right-hand side of Eq. (11) corresponds to partial summation of the iteration series obtained from Eq. (11). It is possible to perform such partial summation due to the presence of the operator L_0 on the right-hand side of the operator L_1 and to the definition of the Green function (7).

The system of equations (9) and (11) is closed and forms a basis for the stochastic theory of filtration. Generalization to the multidimensional case does not change the form of Eqs. (9) and (11).

2. As a specific application of Eqs. (9) and (11), we will calculate the output $q(x)$:

$$q(x) = k(x) \frac{dP(x)}{dx}. \tag{12}$$

Substituting expression (9) into Eq. (12), we will have the following relation for the output:

$$q(x) = k(x) \frac{dP_0(x)}{dx} + \frac{k(x)}{l} \int_0^l dx_1 G_x(x, x_1) T(x_1) P_0(x_1). \tag{13}$$

Using Eq. (11) and performining the iteration procedure in Eq. (13) with account for terms quadratic in the permeability fluctuation, we obtain the following expression for the output:

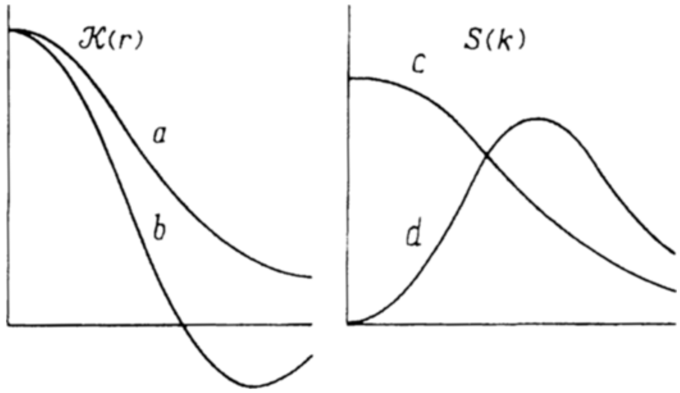

Fig. 1. Specific features of correlation functions and the corresponding spectral functions of the first (curves a and c) and second (curves b and d) kind.

$$q(x) = q_0 \left\{ 1 + \frac{k_1(x)}{k_0} + \frac{1}{k_0 l} \int_0^l dx_1 G_x(x, x_1) \frac{dk_2(x_1)}{dx_1} - \right.$$

$$- \frac{1}{k_0^2 l} \int_0^l dx_1 G_x(x, x_1) k_1(x_1) \frac{dk_1(x_1)}{dx_1} + \frac{k_1(x)}{k_0^2 l} \int_0^l dx_1 G_x(x, x_2) \frac{dk_1(x_1)}{dx_1} +$$

$$\left. + \frac{1}{k_0^2 l^2} \int_0^l dx_1 \int_0^l dx_2 G_x(x, x_1) G_{x_1}(x_1, x_2) \frac{dk_1(x_1)}{dx_1} \frac{dk_1(x_2)}{dx_2} \right\}, \qquad (14)$$

where $q_0 = k_0(dP_0(x)/dx)$ is the output of a homogeneous stratum.

Using the Green function in Eq. (7)

$$G(x, x_1) = \begin{cases} (x_1 - l)\, x/l, & x \le x_1, \\ (x - l)\, x_1/l, & x \ge x_2, \end{cases}$$

after lengthy but simple transforms in Eq. (14) and averaging over an ensemble of realizations of the random function $k(x)$, we finally obtain the following expression for the output [5]:

$$\frac{\langle q \rangle}{q_0} = 1 - \frac{D}{k_0^2} \left[1 - \frac{1}{Dl^2} \int_0^l dx \int_0^l dx_1 K(x, x_2) \right], \qquad (15)$$

where $K(x, x_1) = \langle k_1(x) k_1(x_1) \rangle$ is the correlation function of the permeability fluctuations.

In approximating a correlation function, monotonically decreasing functions are generally used: the Laplace function $\exp(-|x|/a)$, the Karman function $(1 + |x|/a)\exp(-|x|/a)$, and the Gauss function $\exp(-x^2/a^2)$. Their fundamental characteristics are identical, i.e., they attenuate at distances of the order of the correlation radius a; the spectral density $S(k)$ also decreases at a distance equal to the correlation radius and, what is quite important, $S(k = 0) > 0$. However, as shown in [7, 8], in a number of cases the class of monotonically decreasing functions is not suitable for modeling inhomogeneities and must be extended. In particular, it is shown in [7] that the Laplace function leads to divergent terms in fluctuation corrections to the frequencies of electromagnetic waves in inhomogeneous thin films. The need to extend the class of correlation functions was first pointed out in [9], where the following classification of spectral functions was given:

$$\lim_{V \to \infty} \int_V K(r)\, d^3x = (2\pi)^3 S(0) \begin{cases} > 0 & \text{the first type}, \\ = 0 & \text{the second type}. \end{cases}$$

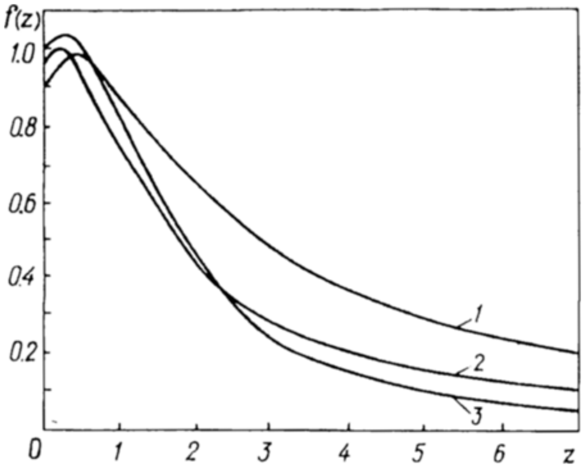

Fig. 2. Calculated values of the functions $f_0(z)$ (curve 1), $f_1(z)$ (curve 2), and $f_2(z)$ (curve 3).

The specific features of correlation functions and the corresponding spectral functions are graphically shown in Fig. 1. Curves *a* and *b* correspond to correlation functions of the first ($S(0) > 0$) and second ($S(0) = 0$) kind; curves *c* and *d* correspond to the spectral functions of the correlation functions of the first and second kind, respectively.

A correlation function of the second kind is valid for specific strata. For example, the empirical normalized correlation function of the permeability for the *DI* stratum of the Bavlinsk oil deposit has a negative half-wave [5] and corresponds to correlation functions of the second kind.

As specific functions for calculating the output, we will consider a class of nonmonotonic functions of the form [10]

$$K(x, x_1) = \left[1 - (x - x_1)^{2n}/a^{2n}\right] \exp\left[-(x - x_1)^2/a^2\right]. \tag{16}$$

Substituting Eq. (16) into Eq. (15), we obtain the following expression for the output:

$$\frac{\langle q \rangle}{q_0} = 1 - \frac{D}{k_0^2}\left(1 - \frac{1}{D} f\left(\frac{l}{a}\right)\right), \tag{17}$$

and the specific form of the function $f(z)$ depends on the choice of the correlation function $K(x, x_1)$. The calculations lead to the following results:

$$f_0(z) = z^{-2}\left[\sqrt{\pi}\, z\, \mathrm{erf}(z) + \exp(-z^2) - 1\right], \tag{18}$$

$$f_1(z) = \frac{\sqrt{\pi}}{2} z\, \mathrm{erf}(z), \tag{19}$$

$$f_2(z) = z^{-2}\left[\frac{\sqrt{\pi}}{4} - z\, \mathrm{erf}(z) - \exp(-z^2) + 1\right] - \frac{1}{2}\exp(-z^2), \tag{20}$$

where $f_0(z)$ corresponds to the Gauss function [5], and $f_1(z)$ and $f_2(z)$ to the functions (16) for $n = 1$ and $n = 2$, respectively. Figure 2 presents numerical calculations of the functions (18), (19), and (20), where curves 1, 2, and 3 correspond to the functions f_0, f_1, and f_2. As is seen, the values of the functions f_1 and f_2, which correspond to correlation functions of the second kind, are smaller in the region $l > a$ than for correlation functions of the first kind.

3. In conclusion it should be noted that in contrast to correlation functions of the first kind, which depend on one parameter, namely, the correlation radius a, correlation functions of the second kind are two-parameter

and depend on the correlation radius a on the "depth" index of the negative half-wave n. This provides wider possibilities in explaining experimental data.

NOTATION

$k(x)$, random inhomogeneous permeability; $P(x)$, pressure; $g(x)$, output; D, permeability dispersions; $S(k)$, spectral permeability function.

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